MATRIX ROUTINE APPLICATION PROGRAM: SIMPLEX PROBLEM

The LP program solves the linear maximisation and minimisation problems with a slightly modified Simplex method. Slack variables are not explicitly used, to save memory space. The Simplex maximisation problem can be described as follows:

A number of positive variables are submitted to a number of constraints of the form:

$g_i(x_1, x_2, \ldots, x_n) \leqslant b_i$ for $i=1,2,\ldots,m$

where the g_i are linear functions.

The problem stated is to find a set of solutions $(x_1, x_2, \ldots, x_n)$ for the constraints g_i so as to maximize a given linear function $f(x_1, x_2, \ldots, x_n)$.

A sample problem:

$$\begin{aligned} x_1 + 3x_2 &\leqslant 300 \\ x_1 + x_2 &\leqslant 160 \\ 2x_1 + x_2 &\leqslant 170 \end{aligned}$$

maximize $f(x_1, x_2) = 20x_1 + 30x_2$

The problem can be represented in matrix form as:

$$\left[\begin{array}{cc|c} 1 & 3 & 300 \\ 1 & 1 & 160 \\ 2 & 1 & 170 \\ \hline 20 & 30 & 0 \end{array}\right]$$

Thus, a simplex problem with m constraints and n variables can be mapped on an (m+1)x(n+1) matrix.

The LP routine published here requires such an input matrix. The bottom row stands for the function to maximize.

REMARK: If $f(x_1, x_2, \ldots, x_n)$ contains a constant it must be entered as the bottom right element of the input matrix but with opposite sign!

e.g. if $f(x_1, x_2) = 20x_1 + 30x_2 + 100$, the bottom row of the input matrix becomes: $\lfloor 20 \quad 30 \mid -100 \rfloor$

LPI and LPO are initialisation routines for the general input -output routine for dimensioned arrays I have written. You can always use your own favorite input scheme instead. Take care to initialize the following data before a run of LP:

R07: starting address of the matrix

R08: number of columns=number of variables +1

R09: number of rows=number of constraints +1

Flag 2: clear for maximisation, set for minimisation.

Size requirements: To keep track of the basic and slack variables involved in the algorithm ,an extra row of data is used.The starting address of the input matrix must be 16 or higher.

Back to the sample problem.A run of LP on the input matrix yields the following matrix:

$$\begin{bmatrix} -0.2 & 0.4 & 0 \\ -0.4 & -0.2 & 32 \\ 0.6 & -0.2 & 0 \\ 42 & 86 & 3420 \end{bmatrix}$$

The upper 3 rows are hardly of any further interest to the user.The bottom row stands for the solution as follows: $x_1=42$ and $x_2=86$ give a maximum of 3420 for the function $f(x_1,x_2)$.This can be verified by substituting this solution into the constraints g_i.

For the minimisation problem,the constraints are of the form $g_i(x_1,x_2,\ldots,x_n) \geqslant b_i$.The objective function f must be minimised.As described above,the problem can again be mapped on a matrix.

The LP routine essentially solves the maximisation problem. To solve the minimisation problem,it is transformed into the former one.This is achieved by first transposing the input matrix,a few changes in the slack variable takeover routine, and finally transposing the matrix again.This is,of course, not the fastest way to solve the minimisation problem,but it saves a lot of extra slack variable registers which are needed when using other methods.Flag 2 is used to select either the first or the second of both problem types.

The routine which transposes the matrix in the 2nd case has been written as a stand alone routine.Here all credits go to JOHN KENNEDY (PPC 918),who wrote an excellent TP program. The version published here only differs from his original version in that it uses the function REGMOVE instead of the PPC ROM "BM".The TP routine transposes any matrix specified by R07,R08 and R09,like the matrix routines.The contents of R08 and R09 are exchanged by TP,of course.

Technical details:

Data registers:

RØ7: starting address of matrix

RØ8: number of columns=number of variables +1

RØ9: number of rows=number of constraints +1

R1Ø: ISG constant for row selection

R11: ISG pointer to constraint constants

R12: ISG pointer to objective function coëfficiënts.

R13: save pivot address

R14: ISG constant to extra row.

alpha registers M,N and O are used for scratch and loop control.

PPC ROM ROUTINES USED:

"BC","BX","M2","M3","M4","M5"

flag 2: used

display mode:not used

angular mode:not used

A good sample problem for those intending to analyze the system is the following:

$$x_1 + 2x_2 + x_3 \leqslant 2$$
$$2x_1 + 3x_2 + x_3 \leqslant 3$$
$$x_1 + x_2 + 4x_3 \leqslant 4$$

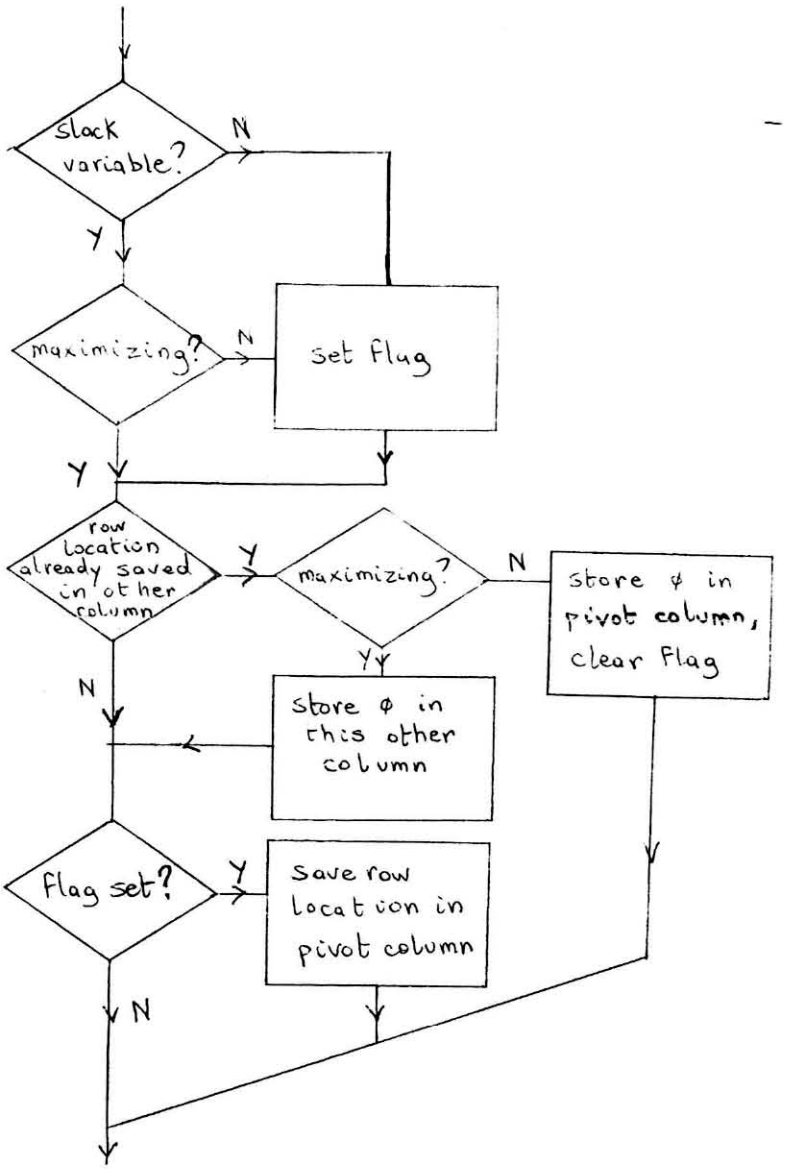
Maximize $18x_1 + 24x_2 + 16x_3$

The solution is $x_1 = 8/7$ $x_2 = Ø$ $x_3 = 5/7$ maximum=32

PHILIPPE ROUSSEL PPC4367BC1

Perhaps the method used needs some more explanation. It can be understood by observing the following: at the start of the solution method, all slack variable coefficients are either set to 1 or 0. When the matrix column pertaining to a slack variable is changed for the first time, the column pertaining to one of the main variables becomes a unity base vector (such as <0,0,1,0>). Such columns are not explicitly needed. They simply indicate that the main variable related to it "has been taken over" by a slack variable. By keeping track of this takeover process in an extra matrix row, the unit vectors are no longer explicitly necessary in the system matrix. Especially for larger systems, the number of storage locations needed can be reduced using the above method, which is certainly interesting when implementing the Simplex method on small systems like the HP-41C*.

slack variable take over IN LP.RPN S/R @ LINE 129.




```
RCL 14
- - + - -
STO 0
CLX
RCL IND 0
X ≠ ϕ ?
FS? ϕ2
SF ϕ5
RDN
STO 13
XROM^T M2
RCL 14
SIGN
RCL 13
GTO ϕϕ
LBL ϕ7
RCL IND L
X <> Y
X ≠ Y ?
GTO ϕϕ
CLX
FS? ϕ2
GTO ϕ5
STO IND L
GTO ϕ6
LBL ϕϕ
ISG L
GTO ϕ7
LBL ϕ6
RCL 13
LBL ϕ5
FS? C ϕ5
STO IND 0
- - - - - - -
1
STO 0
LBL ϕ2
```

Sample procedure

$$\left[\begin{array}{cccccc|c}
1 & 2 & 1 & 1 & 0 & 0 & 2 \\
2 & 3 & 1 & \phi & 1 & \phi & 3 \\
1 & 1 & 4 & 0 & 0 & 1 & 4 \\
 & \uparrow & & \text{slack} & \text{variables} & & \\
18 & 24 & 16 & \phi & 0 & \phi & \phi
\end{array}\right]$$

largest objective function coeff.

Find smallest a_{in} / a_{i2} with $a_{in} \geq 0$ & $a_{i2} > 0$

$$\left[\begin{array}{ccccccc}
1/2 & 1 & 1/2 & 1/2 & \phi & \phi & 1 \\
1/2 & \phi & -1/2 & -3/2 & 1 & \phi & \phi \\
1/2 & \phi & 7/2 & -1/2 & \phi & 1 & 3 \\
6 & \phi & 4 & -12 & \phi & \phi & -24
\end{array}\right]$$

r_1 / a_{12}
$r_2 - a_{22} \cdot r_1$
$r_3 - a_{32} \cdot r_1$
$r_4 - a_{42} \cdot r_1$

$\uparrow x_2$

Find smallest a_{in} / a_{i1} ...

$$\left[\begin{array}{ccccccc}
\phi & 1 & 1 & 2 & -1 & \phi & 1 \\
1 & \phi & -1 & -3 & 2 & \phi & \phi \\
\phi & \phi & 4 & 1 & -1 & 1 & 3 \\
\phi & \phi & 10 & 6 & -12 & \phi & -24
\end{array}\right]$$

r_2 / a_{21}

$r_1 - a_{11} r_2$
$r_3 - a_{31} r_2$
$r_4 - a_{41} r_2$

$\uparrow x_1$

$$\left[\begin{array}{ccccccc}
\phi & 1 & \phi & 7/4 & -3/4 & -1/4 & 1/4 \\
1 & \phi & \phi & -11/4 & 7/4 & 1/4 & 3/4 \\
\phi & \phi & 1 & 1/4 & -1/4 & 1/4 & 3/4 \\
\phi & \phi & \phi & 7/2 & -13/2 & -10/4 & -\frac{63}{2}
\end{array}\right]$$

$x_2 = 1/4$

$x_1 = 3/4$

$\boxed{\text{is not solution}}$ $x_3 = 3/4$

indicated by positive objective function coeff. left.

$\uparrow x_3$

$$\left[\begin{array}{ccccccc}
\phi & 4/7 & \phi & 1 & -3/7 & -1/7 & 1/7 \\
1 & 11/7 & \phi & \phi & 4/7 & -1/7 & 8/7 \\
\phi & -1/7 & 1 & \phi & -1/7 & 2/7 & 5/7 \\
\phi & -2 & \phi & \phi & -8 & -2 & -32
\end{array}\right]$$

$x_2 = \phi$

$x_1 = 8/7$

$x_3 = 5/7$

$\rightarrow \max = 32$

```
01♦LBL "LPO"
 02 CF 10
 03 RCL 09
 04  E3
 05 /
 06 RCL 09
 07 +
 08 GTO 00

09♦LBL "LPI"
 10 "C↑V↑B?"
 11 PROMPT
 12 SF 10
 13 STO 07
 14 RDN
 15 1
 16 +
 17 STO 08
 18 X<>Y
 19 1
 20 +
 21 STO 09

22♦LBL 00
 23 STO 01   ✓ As04
 24 2
 25 STO 00   ✓ Pvt02
 26 RCL 08
 27 STO 02   ✓ As08
 28 RCL 07
 29 "a"
 30 CF 01
 31 CF 04
 32 FS?C 10  For LPIN S/R
 33 GTO "IN"
34 GTO "OUT"
```

```
'SIM' LINE 01
35♦LBL "LP"
 36 CF 10
 37 FS? 02
 38 XEQ "TP"  T
 39 RCL 07
 40 DSE X
 41 RCL 08
 42 RCL 09
 43 1
 44 -
 45 *
 46 +
 47 RCL X
 48  E3
 49 /
 50 ST+ Z
 51 +
 52 RCL 08
 53 1
 54 -
 55  E-3
 56 *
 57 ST+ Y
 58 X<>Y
 59 STO 12
 60 LASTX
 61 +
 62 +
 63 RCL 08
 64 +
 65 STO 14
 66 1
 67 +
 68 XROM "BC"  Z
 69 X<>Y
 70 RCL 08
 71  E5
 72 /
 73 +
 74 STO 10
 75 RCL 08
 76 +
 77 STO 11
 78 ISG 12

 79♦LBL 15
 80 RCL 12
 81 XROM "BX"  X
 82 X<>Y
 83 X<=0?
 84 GTO 03
 85 RCL [  00→16
 86 INT
 87 XROM "M4"  4
 88 RCL 10
 89 +
 90 STO [  00→16
 91 RCL 08
 92  E3
 93 /
 94 +
 95 STO \  01→17
 96 RCL 11
 97 STO ]  02→18
 98 CLX
```

```
'SIM' LINE 125
99♦LBL 01
 100 RCL IND [  00→16
 101 X<=0?
 102 GTO 00
 103 RCL IND ]  02→18
 104 X≠0?
 105 GTO 05
 106 STO ]  02→18
 107 GTO 06

108♦LBL 05
 109 /
 110 X<Y?
 111 GTO 00

112♦LBL 06
 113 RCL [  00→16
 114 STO 13

115♦LBL 00
 116 X<>Y
 117 ISG [  00→16
 118 ""
 119 ISG ]  02→18
 120 GTO 01
 121 1
122 X<> IND 13
 123 1/X
 124 RCL 13
 125 INT
126 XROM "M4"  4
 127 RCL 14
 128 +
 129 STO ]  02  SLACK VARIABLE TAKEOVER, SEE FLOWCHART
 130 CLX
131 RCL IND ]  02→18
 132 X≠0?
 133 FS? 02
 134 SF 05
 135 RDN
 136 STO 13
137 XROM "M2"  2
 138 RCL 14
 139 SIGN
 140 RCL 13
 141 GTO 00

142♦LBL 07
143 RCL IND L
 144 X<>Y
 145 X≠Y?
 146 GTO 00
 147 CLX
 148 FS? 02
 149 GTO 05
150 STO IND L
 151 GTO 06

152♦LBL 00
 153 ISG L
 154 GTO 07

155♦LBL 06
 156 RCL 13
```

```
'SIM' LINE 215
157♦LBL 05
 158 FS?C 05
 159 STO IND ]  02→18
 160 1
 161 STO ]  02→18

162♦LBL 02
 163 RCL ]  02→18
 164 RCL 13
 165 X=Y?
 166 GTO 00
 167 0
168 X<> IND \  01→17
 169 CHS
 170 X≠0?
171 XROM "M3"  3

172♦LBL 00
 173 ISG ]  02→18
 174 ""
 175 ISG \  01→17
 176 GTO 02
 177 GTO 15

178♦LBL 03
 179 -1
 180 RCL 09
181 XROM "M2"  2
 182 FC? 02
 183 RCL 12
 184 FS? 02
 185 RCL 11
186 XROM "BC"  Z
 187 RCL 14
 188 1
 189 +
 190 STO 13
 191 DSE 12

192♦LBL 04
193 RCL IND 13
 194 X=0?
 195 GTO 00
 196 RCL 08
197 XROM "M5"  5
 198 RCL 13
 199 RCL 14
 200 -
 201 RCL 12
 202 +
203 RCL IND Y
204 X<> IND Y
205 STO IND Z

206♦LBL 00
 207 ISG 13
 208 GTO 04
 209 FS? 02
 210 XEQ "TP"  T
 211 END
 329b=47R+0b
```

```
01♦LBL "TP"
 02 RCL 07   Start Adr.   17
 03 ENTER↑
 04 STO 01   ←07   17
 05 STO 02   ←07   17
 06 STO 04   ←07   17
 07 RCL 09   + Rows+1   5
 08 ST+ 04   ← As Int   22
 09 +   22
 10  E3   For Isg.   .022
 11 /
 12 +   17.022
 13 RCL 08   Columns+1   5
 14 ST- 04   17
 15 ST- 04   17
 16 STO 03   Col+1   5
 17 RCL 09   Rows+1   5
 18 *   25
 19 ST+ 04   37
 20  E6   .000029
 21 /
 22 +   17.022029
 23 REGMOVE
 24 ISG 04   38

X25♦LBL 01   (Skips By Force.)
 26 RCL 09   Row+1   5
 27 STO 00   5
 28 ST+ 02   22
 29 RCL 02   22
 30 RCL 03   Col+1   5

 31♦LBL 02
 32 RCL IND Y
 33 STO IND 01
 34 RDN
 35 ISG 01
 36 ""
 37 +
 38 LASTX
 39 DSE 00
 40 GTO 02
 41 DSE 03
 42 GTO 03
 43 RCL 08
 44 X<> 09
 45 STO 08
 46 RTN

47♦LBL 03
 48 RCL 09
 49 STO 00
 50 RCL 04
 51 STO 05
 52 1
 53 ST- 00
 54 +
 55 STO 06
```

```
56♦LBL 04
 57 RCL 05
 58 RCL 06
 59  E3
 60 /
 61 +
 62 RCL 03
 63  E6
 64 /
 65 +
 66 REGMOVE
 67 DSE 05
 68 RCL 03
 69 ST- 05
 70 ST- 06
 71 DSE 00
 72 GTO 04
 73 2
 74 ST+ 04
 75 GTO 01
 76 END
108b=15R+3b
```

R:07 Start Adr.
R:08 Columns+1
R:09 Rows+1
R:10 Isg For Row
R:11 Isg Pointer For Constraint Constants.
R:12 Isg Pointer For Objective Func Coefficients
R:13 Save Pivot Adr.
R:14 Isg Constant For Extra Row.

SYNTHETICS

P.J. Roussel's Version of 'SIM' Called 'LP' For Linear Programming.

Non-Syntho' (RPN) version of 'LP', renamed 'SIM'

```
 01♦LBL "SIM"
CLX  "MIN=1"  PROMPT
X≠0?  SF 02  FS? 02
XROM "T"  RCL 07  DSE X
RCL 08  RCL 09  1  -  *
+  RCL X  1 E3  /  ST+ Z
+  RCL 08  1  -  1 E-3
*  ST+ Y  X<>Y  STO 12
LASTX  +  +  RCL 08  +
STO 14  1  +  SIGN  CLX


 40♦LBL 13
STO IND L  ISG L  GTO 13
X<>Y  RCL 08  1 E5  /  +
STO 10  RCL 08  +
STO 11  ISG 12

 54♦LBL 15
RCL 12  STO 00  STO 01
STO 02  RCL IND X
ENTER↑  ENTER↑  RDN



 63♦LBL 08
CLX  RCL IND Z  X>Y?
GTO 10  R↑  X>Y?  GTO 11
RDN

 72♦LBL 09
ISG Z  GTO 08  X<>Y  R↑
GTO 14

 78♦LBL 10
X<>Y  CLX  RCL Z  STO 00
GTO 09

 84♦LBL 11
CLX  RCL T  STO 01  X<>Y
RDN  GTO 09

 91♦LBL 14
X<>Y  X<=0?  GTO 03
RCL 00  INT  RCL 07  -
RCL 08  X<>Y  STO 02
X<>Y  MOD  ST- 02  LASTX
ST/ 02  CLX  X<> 02
X<>Y  ISG Y  " "  ISG X
" "  RCL 10  +  STO 00
RCL 08  1 E3  /  +
STO 01  RCL 11  STO 02
CLX

125♦LBL 01
RCL IND 00  X<=0?
GTO 00  RCL IND 02  X≠0?
GTO 05  STO 02  GTO 06


134♦LBL 05
/  X<Y?  GTO 00
138♦LBL 06
RCL 00  STO 13

141♦LBL 00
X<>Y  ISG 00  CLD
ISG 02  GTO 01  1
X<> IND 13  1/X  RCL 13
INT  RCL 07  -  RCL 08
X<>Y  STO 02  X<>Y  MOD
ST- 02  LASTX  ST/ 02
CLX  X<> 02  X<>Y  ISG Y
" "  ISG X  " "  RCL 14
+  STO 02  CLX
RCL IND 02  X≠0?  FS? 02
SF 05  RDN  STO 13
RCL 08  *  RCL 07  +
RCL X  RCL 08  ST- Z
SIGN  -  1 E3  /  +
X<>Y

192♦LBL 12
ST* IND Y  ISG Y  GTO 12
RCL 14  SIGN  RCL 13
GTO 00

200♦LBL 07
RCL IND L  X<>Y  X≠Y?
GTO 00  CLX  FS? 02
GTO 05  STO IND L
GTO 06

210♦LBL 00
ISG L  GTO 07

213♦LBL 06
RCL 13

215♦LBL 05
FS?C 05  STO IND 02  1
STO 02

220♦LBL 02
RCL 02  RCL 13  X=Y?
GTO 00  0  X<> IND 01
CHS  X≠0?  XROM "3"


230♦LBL 00
ISG 02  CLD  ISG 01
GTO 02  GTO 15

236♦LBL 03
-1  RCL 09  RCL 08  *
RCL 07  +  RCL X  RCL 08
ST- Z  SIGN  -  1 E3  /
+  X<>Y

252♦LBL 16
ST* IND Y  ISG Y  GTO 16
FC? 02  RCL 12  FS? 02
RCL 11  SIGN  CLX


262♦LBL 17
STO IND L  ISG L  GTO 17
RCL 14  1  +  STO 13
DSE 12

271♦LBL 04
RCL IND 13  X=0?  GTO 00
RCL 08  X<> 08  ST- 08
*  ST+ 08  X<> L  X<> 08
1  -  RCL 07  +  RCL 13
RCL 14  -  RCL 12  +
RCL IND Y  X<> IND Y
STO IND Z

294♦LBL 00
ISG 13  GTO 04  FS? 02
XROM "T"  TONE 9  GE
END
```

NoLABELS.
SEE M3'

```
GTO "3"
 08♦LBL "3"
 09 STO 00
 10 RDN
 11 XEQ 00
 12 X<>Y
 13 XEQ 00
 14 RCL 00
 15 SIGN
 16 RDN
 17 RCL IND Y
 18 LASTX
 19 *
 20 ST+ IND Y
 21 ISG Y
 22 " "
 23 ISG Z
 24 GTO 02
 25 RTN
 26 RCL 08
 27 *
 28 RCL 07
 29 +
 30 RCL X
 31 RCL 03
 32 ST- Z
 33 SIGN
 34 -
 35 1 E3
 36 /
 37 +
 38 RTN
 39♦LBL "4"
```

ST/ND? NOPS.

```
E000EFFF
 E032>E087 86
 E090>E09F 16
 E0D8>E0DF 8
 E153>E18F 61
 E1A3>E1A7 5
 E1E7>E1EB 5
 E211>E217 7
 E24E>E25F 18
 E313>E32F 29
 E445>E44F 11
 E55F>E56F 17
 E5ED>E5EF 3
 E668>E66F 8
 E6A3>E6AF 13
 E764>E76F 12
 E783>E78F 13
 E945>EFFA 1718
                16
XR>16/24
E002/00/E08F SIMPLEX
E004/01/E0A2 'M4
E006/02/E0C0 'M5
E008/03/E262 'MI
E00A/04/E299 'MO SYNTHO'
E00C/05/E2F9 'VW SECTION
E00E/06/E332 'LP ORIGINALS
E010/07/E192 'BC
E012/08/E1AA 'BX
E014/09/E1EE 'M2
E016/10/E21A 'M3
E018/11/E0E2 'TRNS
E01A/12/E452 'SIMPLEX.
E01C/13/E772 'Z BC
E01E/14/E57A 'TRNS
E020/15/E5F2 '2 M2
E022/16/E602 '3 M3
E024/17/E633 '4 M4
E026/18/E651 '5 M5
E028/19/E672 'X BX
E02A/20/E6B2 'SI INPUT
E02C/21/E6E9 'SO OUTPUT
E02E/22/E74A 'V VIEWXY.
E030/23/E792 'SIMPLEX.
```

SIM.RPN ↑
NoSYNTHETICS,
STRAIGHT LINE
VERSION. EXCEPT FOR
'T' & '3'

XROM'T' HAS '77 LINES
+ 2 CALLS TO X-FUNCTIONS
'REGMOVE' (XROM 25,35)